PA1140: Waves and Quanta

Unit 4: Atoms and Nuclei

Tipler 6th ed, Chapters 36 (36-1 to 36-2) & 40

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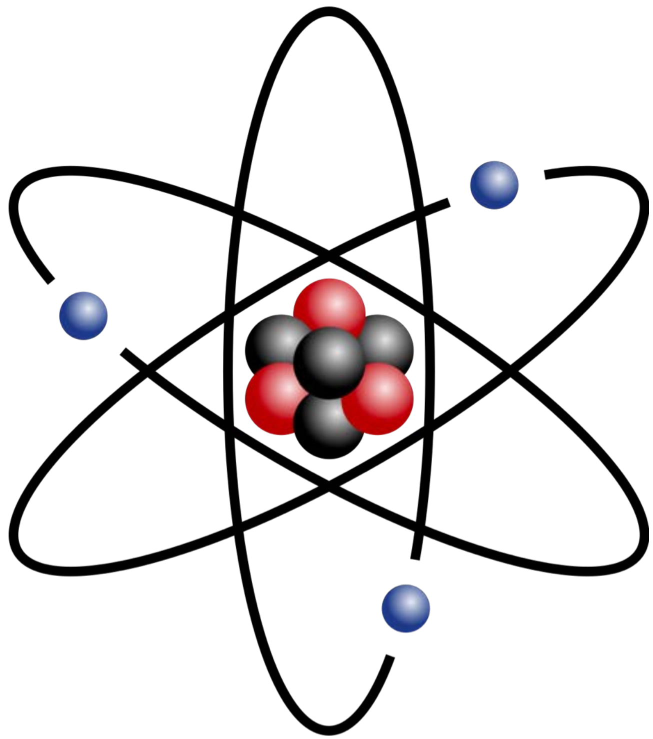


Dr Eloise Marais (Michael Atiyah Annex, 101)

Lecture 1 Recap

Atomic Spectra and the Bohr Model

Rydberg-Ritz formula:

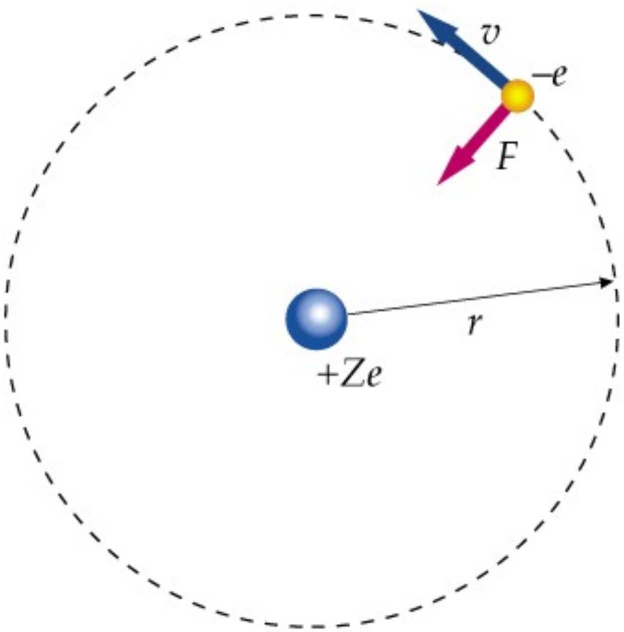
$$\frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

where: n_1 and n_2 are integers

$n_1 > n_2$

$R = 1.096776 \times 10^7\ \mathrm{m}^{-1}$ for hydrogen

The Bohr Model:



v: speed

$-e$: charge of electron

F: attractive electron force

r: radius

$+Ze$: charge of nucleus

Atomic Spectra and the Bohr Model

Energy for a Circular Orbit:

$$E = -\frac{1}{2}\frac{kZe^2}{r}$$

Bohr's Three Postulates:

1. Electron can move only in circular orbits called **stationary states**

2. Electron can make transition from high to low energy orbit leading to emission of photon

3. Only orbits allowed if angular momentum (mvr) of electron is an integer multiple of $h/2\pi = \hbar$ (quantized or discreet values)

$$mv_n r_n = n\hbar$$

Atomic Spectra and the Bohr Model

Photon frequency:

$$f = \frac{E_i - E_f}{h} = \frac{\Delta E}{h}$$

Photon wavelength:

$$\lambda = \frac{c}{f} = \frac{hc}{E_i - E_f}$$

Energy of orbit of hydrogen atom:

$$E_n = -Z^2 \frac{E_0}{n^2}$$

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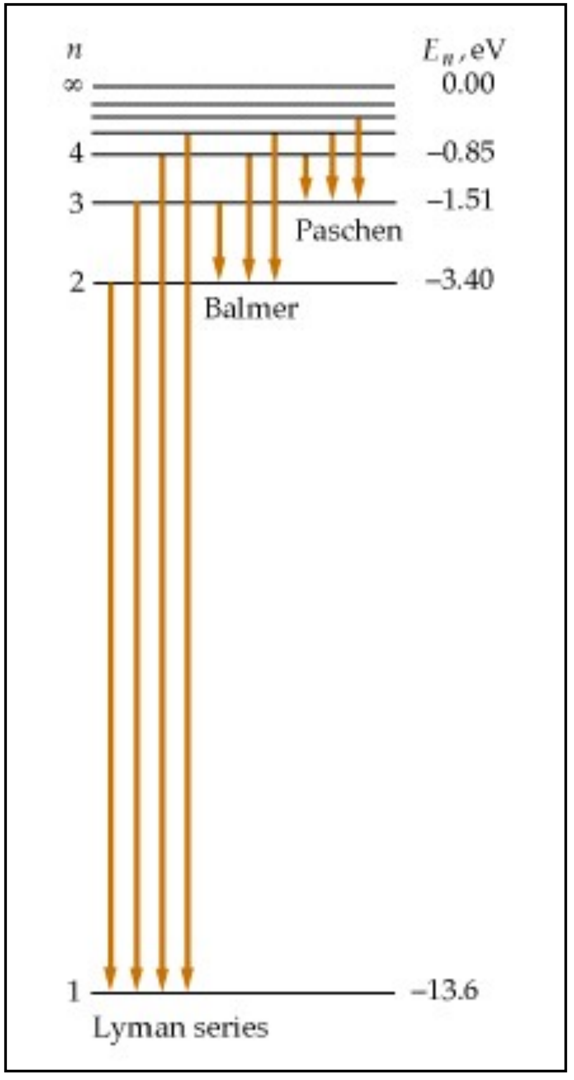
Practice Problem: Atomic Spectra

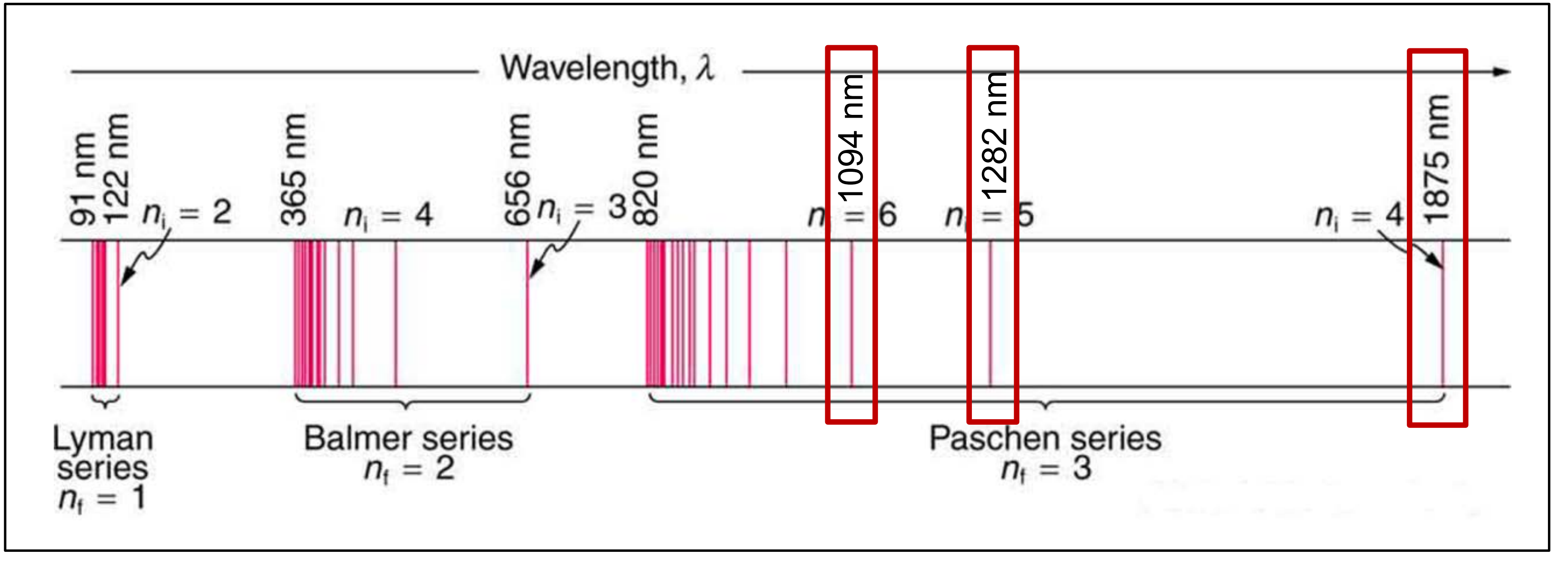
Hydrogen Energy-level diagram



a) Find the photon energy and wavelength for the series limit (shortest wavelength) in the Paschen series (n_2 = 3).

b) Calculate the wavelengths for the three longest wavelengths in this series and indicate their positions on a horizontal linear scale.
The Rydberg constant for hydrogen is 1.097776×10^7 m^{-1}.

Let's check our answers against the observed spectrum:



Practice Problem: Ionization Energy

Calculate the ionization energy for:

a) the hydrogen atom

b) He^{+} (the second element in the periodic table)

c) Li^{2+} (the third element in the periodic table)

Practice Problem: Ionization Energy

Calculate the ionization energy for:

a) the hydrogen atom
Answer: 13.6 eV

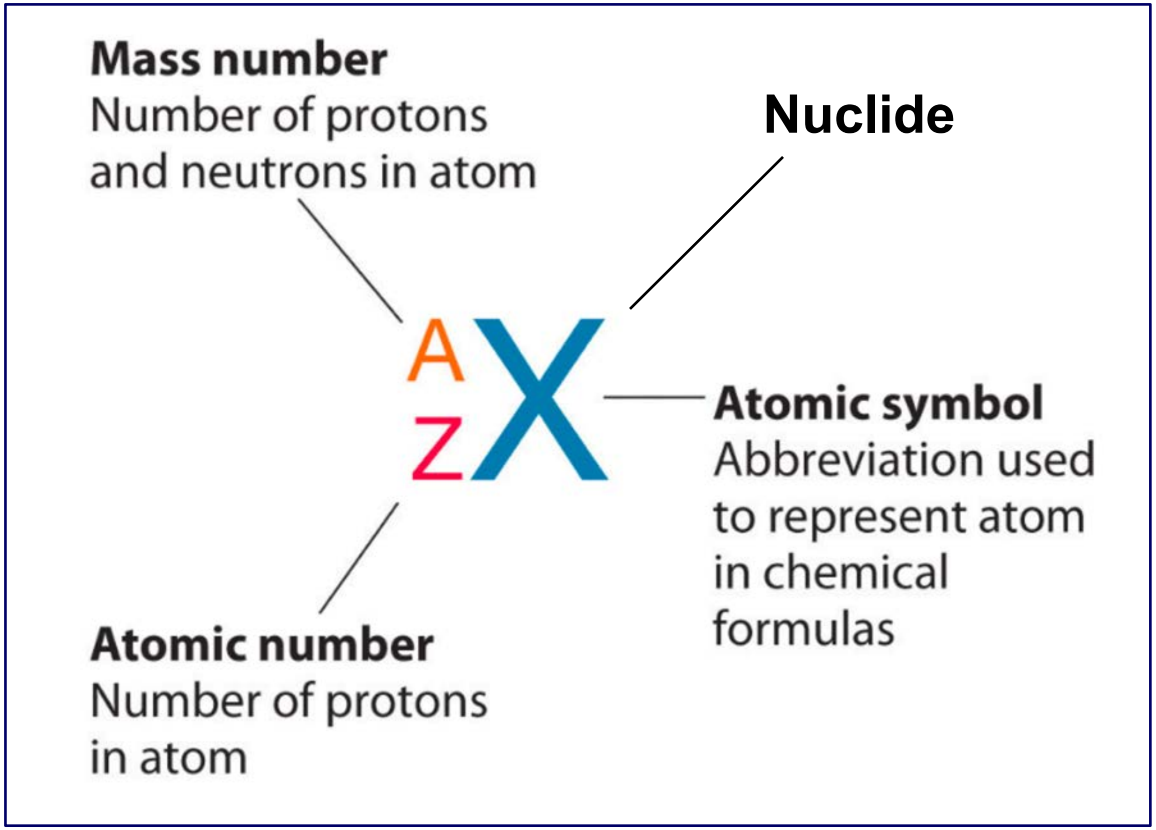
b) He^{+} (the second element in the periodic table)
Answer: 54.4 eV

c) Li^{2+} (the third element in the periodic table)
Answer: 122.4 eV

NUCLEAR PHYSICS

Relevant Terminology and Concepts

- **Nucleus**: includes protons and neutrons (masses are about equal)
- Number of **protons**: **Z** or atomic number
- Number of **neutrons**: **N**
- **Nucleons**: Protons and neutrons in a nucleus
- Number of **nucleons**: **A** or nucleon/mass number (A = N + Z)
- **Mass of nuclei** = mass of a nucleon times A
- **Nuclide**: a nuclear species
- **Atomic symbol**: unique name for an atom (H for hydrogen)
- **Isotope**: 2 or more nuclides with the same Z, but different N (and A)



$$A = N + Z$$

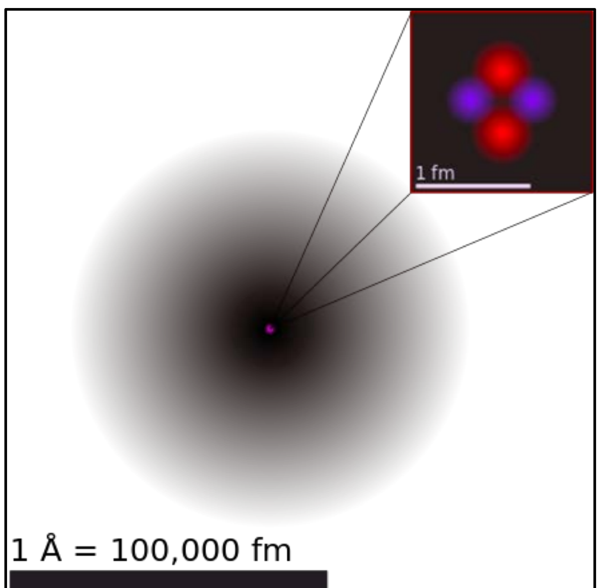
Isotopes of hydrogen:

^{1}H	^{2}H	^{3}H
Protium	Deuterium	Tritium

Strong Nuclear Force

- Nucleons exert strong attractive forces on other nucleons
- Also called the **hadronic force**
- Stronger than electrostatic force of repulsion between protons
- Much stronger than gravitational forces between nucleons (negligible)

Nucleus of He



- About the same between 2 neutrons, 2 protons, and a neutron and proton
- Decreases rapidly with distance
- Negligible for nucleons a few femtometers apart (1 fm = 10^{-15} m)

2 protons stay together due to nuclear force (there is also a repulsive force between two protons).

Nuclear Size, Shape, and Density

Most nuclei are approximately spherical, with **radii** (R):

$$R = R_0 A^{\frac{1}{3}}$$

$\boldsymbol{R_0 \sim 1.2\ \mathrm{fm}}$ (empirical constant)

$\boldsymbol{A = N + Z}$ is the atomic mass

Radius (R) of nucleus is proportional to $A^{1/3}$, so volume (V) is proportional to A

Mass is also approximately proportional to A, so **densities of all nuclei are roughly the same**

Practice Problem: Density of Nuclear Matter

Let's prove that the densities of nuclei are all roughly the same.

N and *Z* of Heavy and Light Nuclei

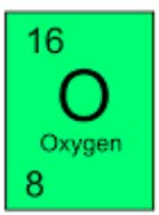
Light nuclei (A < 50):

- stable for $N \approx Z$

Heavier nuclei (A > 50):

- instability caused by electrostatic repulsion between protons
- instability minimized by having more neutrons than protons

Example:



Oxygen-16 has

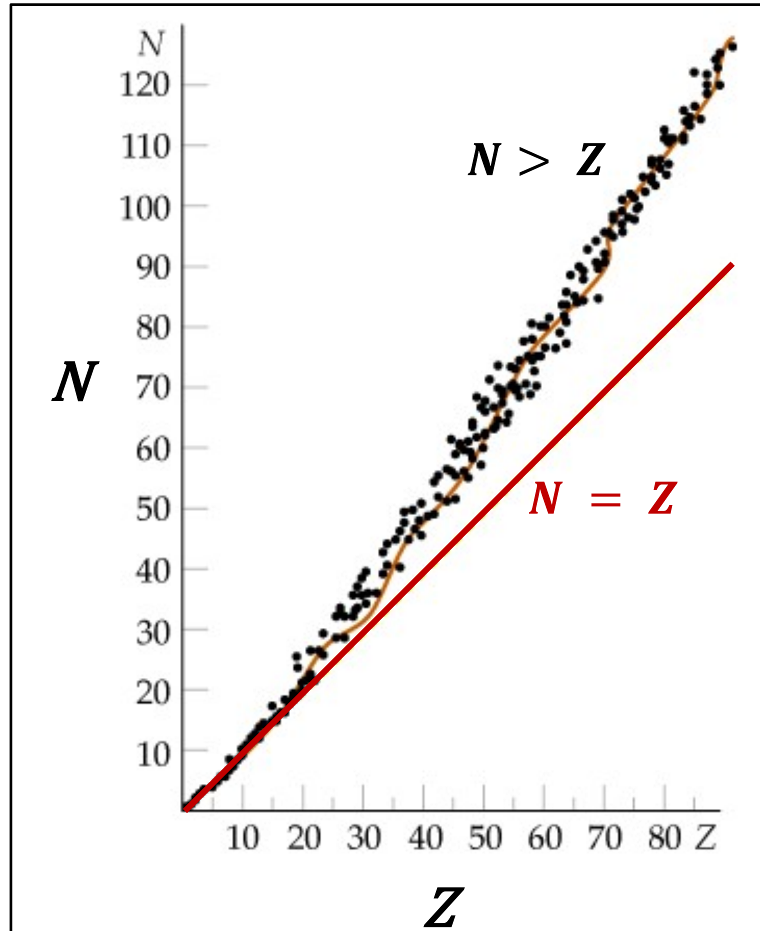
8 Protons
8 Electrons
16-8=8 Neutrons

$$N = Z = 8$$

$$^{238}_{92}\mathrm{U}$$

$$N = 146$$
$$Z = 92$$

Neutrons versus protons



Mass and Binding Energy

Energy is released and mass decreases when 2 or more nucleons fuse to form a nucleon (opposite when break apart)

E_b: **binding energy** (energy needed to break all bonds)

The mass of a nucleus is less than the sum of its parts by E_b/c^2

General formula for E_b:

$$E_b = (ZM_H + NM_N - M_A)c^2$$

M_H: proton mass
M_N: neutron mass
M_A : final atomic mass

Values for M_H and M_A include the masses of the electrons

Formula does not account for mass associated with the binding energies of electrons

Mass and Binding Energy

Atomic and nuclear masses are often given in **unified atomic mass units** (u or **a.m.u.**)

1 u = one-twelfth the mass of carbon-12 (^{12}C)

The rest energy of 1 u (using $E = mc^2$ and m_u = 1.66054 x 10^{-27} kg) is:

$$(1u)c^2 = 931.5 \text{ MeV}$$

Practice Problem: Binding Energy

^{4}He includes 2 protons and 2 neutrons. The mass of ^{4}He measured with a mass spectrometer is **4.002603 u**.

The mass of a proton (^{1}H) is 1.007825 u and that of a neutron is 1.008665 u.

What is the binding energy of the ^{4}He nucleus?

Practice Problem: Binding Energy

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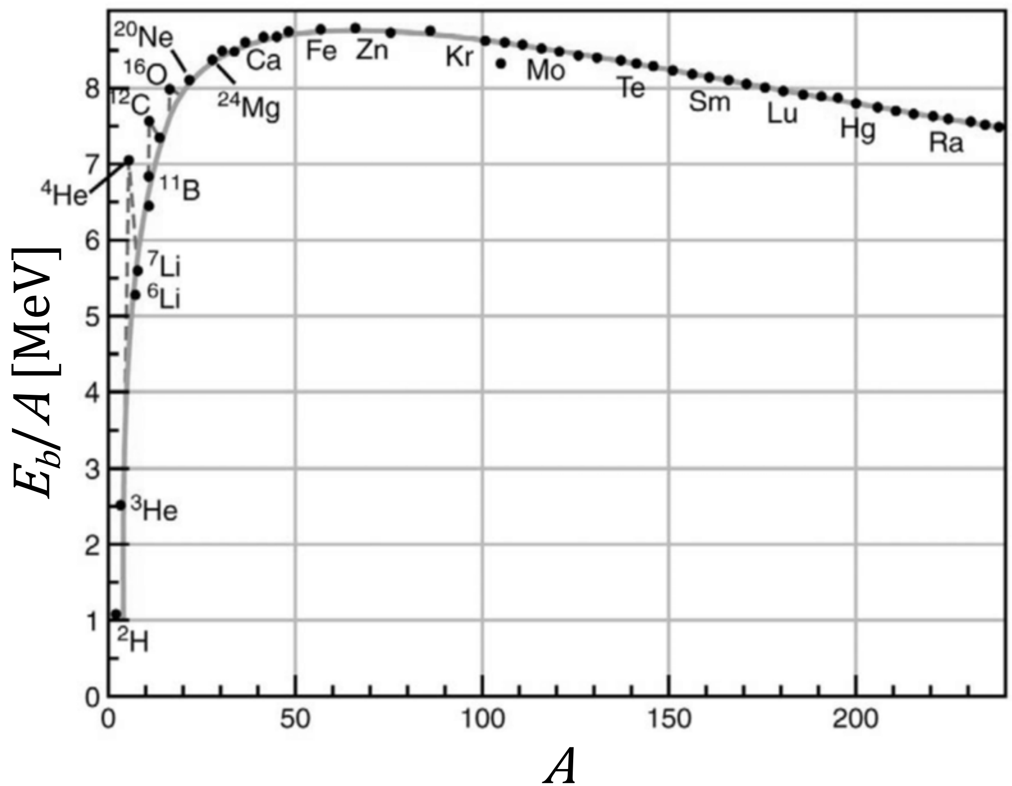
The mass of a proton (^{1}H) is 1.007825 u and that of a neutron is 1.008665 u.

What is the binding energy of the ^{4}He nucleus?

Answer: 28.30 MeV

Mass and Binding Energy

Binding energy vs mass number



A < 20: Increase in nearest neighbours and in number of bonds per nucleon

50 < A < 200: Saturation of nuclear forces

A > 200: Coulomb repulsion so great that nucleus becomes unstable.

A > 300: Nucleus unstable and undergoes spontaneous fission.

RADIOACTIVE DECAY

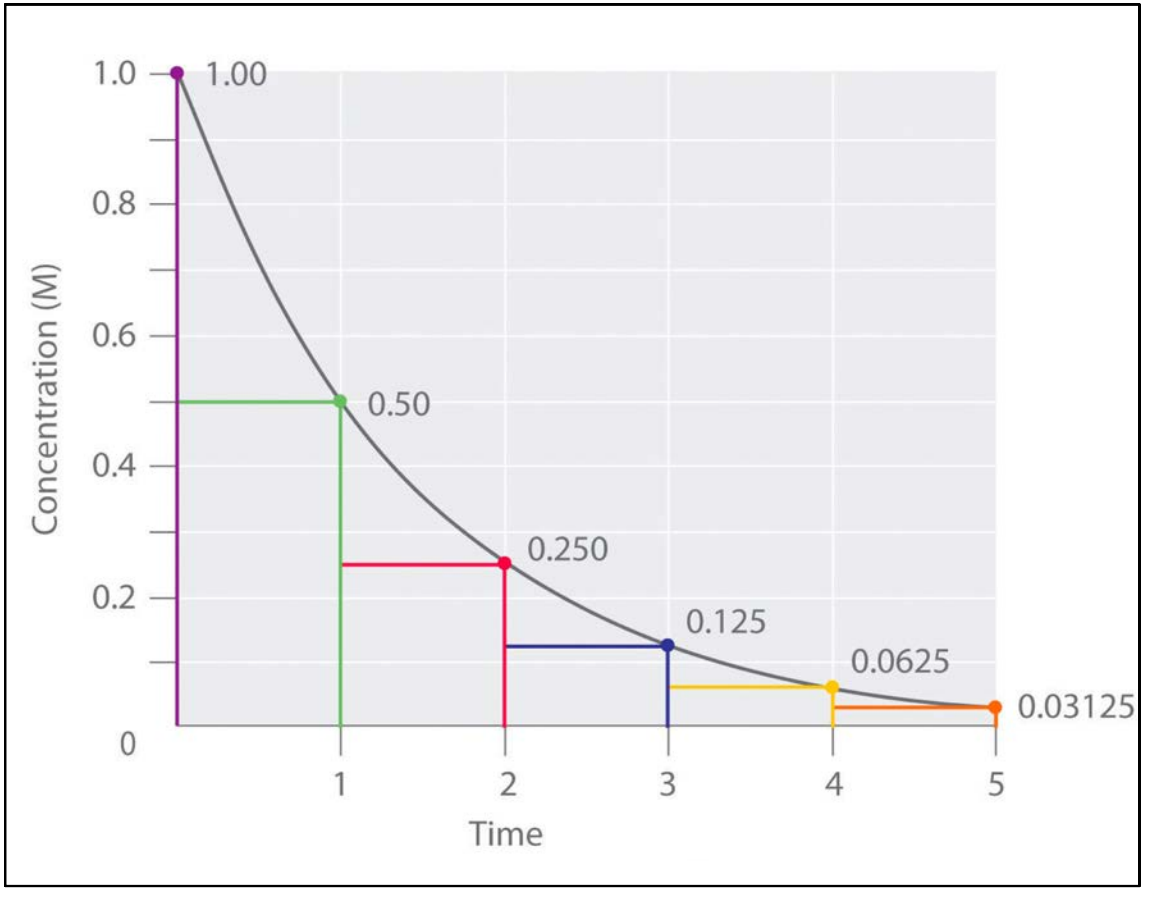
Radioactivity

Relevant Terminology

- **Radioactive nuclei**: decay into other nuclei by emitting particles (photons, electrons, neutrons, α particle)
- Decay can be by $\boldsymbol{\alpha}$ **decay**, $\boldsymbol{\beta}$ **decay**, or $\boldsymbol{\gamma}$ **decay** (we will go through these in detail in Lecture 3)
- **positron**: particle with the same mass as an electron, but positively charged (formed from β decay)
- **Parent nucleus**: original nucleus undergoing decay
- **Daughter nucleus**: nucleus formed as a result of decay

Rate of Decay

Amount of a radioactive sample **decreases exponentially** with time

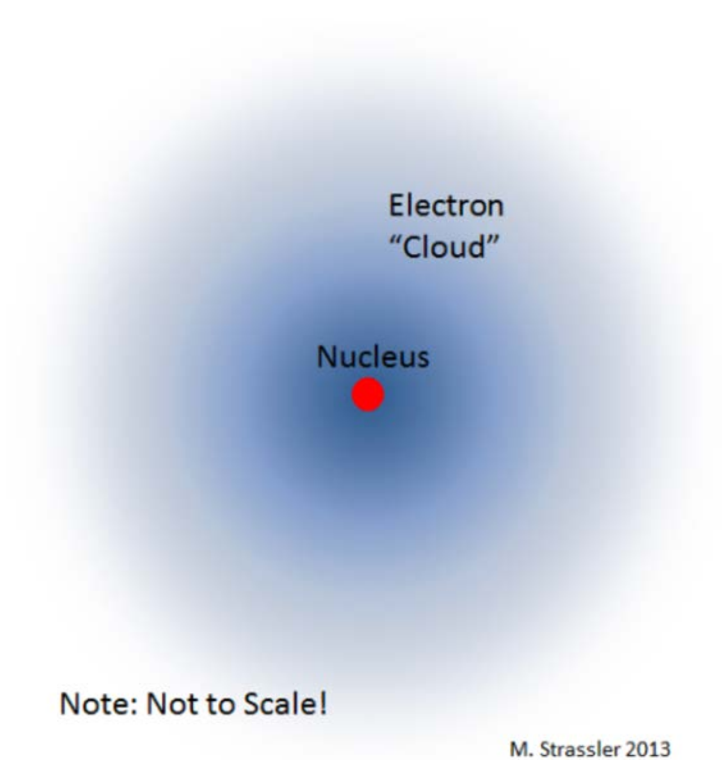


***Statistical process**: curve above represents a population of atoms, rather than individual nuclides*

Rate of Decay

Electrons shield nucleus of one nuclide from the nucleus of others

This means that changes in temperature and pressure have little or no effect on the decay rate
(i.e. **independent of pressure and temperature**)



Electron cloud or shield

Rate of Decay

Consider the radioactive decay of a population of radioactive nuclei

The expression for the change in number N of these nuclei from time t to $t + dt$ is:

$$dN = -\lambda N dt$$

λ is the constant of proportionality or decay constant in s^{-1} (can also be in min^{-1}, day^{-1} etc.) (*not to be confused with wavelength*)

$\frac{dN}{dt} \propto N$ is characteristic of exponential decay

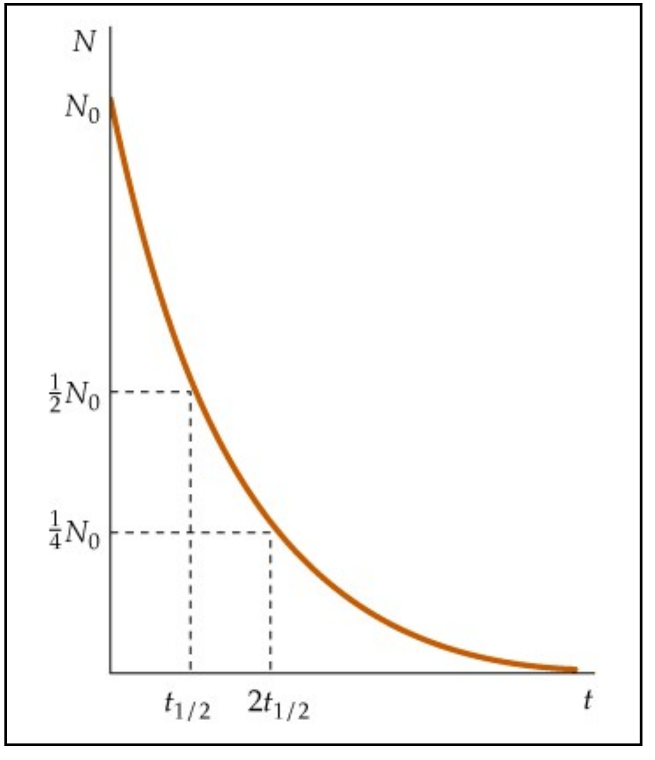
Rate of Decay

Solving for N by integration:

Derivation on board leading to:

$$N = N_0 e^{-\lambda t}$$

Number of particles vs time



Rate of Decay

The number of decays per second is the decay rate or R:

$$R = -\frac{dN}{dt}$$

Remember: $dN = -\lambda N dt$

$$R = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

$$\boxed{R = R_0 e^{-\lambda t}}$$

R is the **decay rate** or **activity of a sample**

$$\boxed{R_0 = \lambda N_0}$$

R_0 is the **decay rate** at $t = 0$

R is determined experimentally