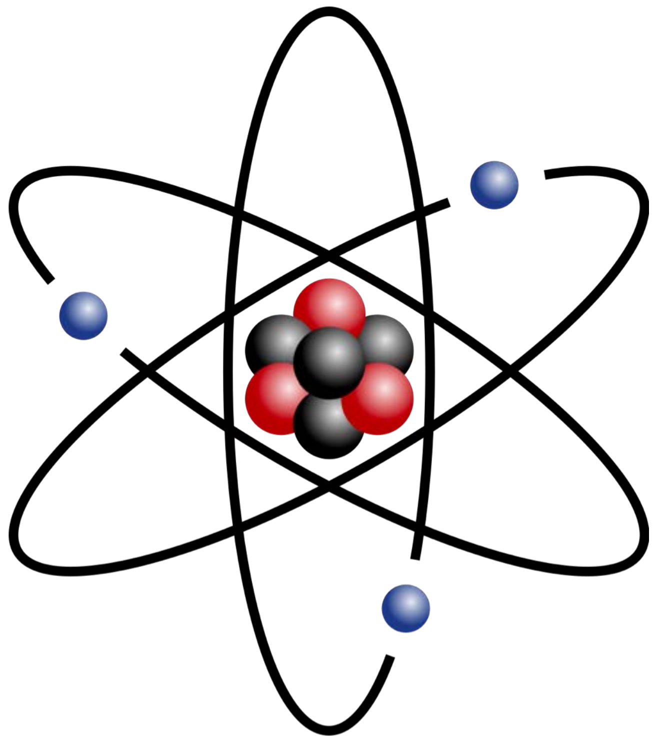
PA1140: Waves and Quanta

Unit 4: Atoms and Nuclei

Tipler 6th ed, Chapters 36 (36-1 to 36-2) & 40

UNIVERSITY OF LEICESTER



Dr Eloise Marais (Michael Atiyah Annex, 101)

Unit 4: Atoms and Nuclei

- The Atom
- Atomic Spectra
- The Bohr Model
- Properties of Nuclei
- Mass and Binding Energy
- Radioactivity
- Nuclear Reactions
- Fission
- Fusion

THE ATOM

Relevant Terminology

- **Atomic number** (Z): number of protons
  matches location on periodic table
  Z = 1 for hydrogen
  Z = 2 for helium
  Z = 3 for lithium etc.

- Number of protons = number of electrons (for uncharged atoms)

- **Nucleus**: comprised of protons and neutrons (radius = 1-10 fm)

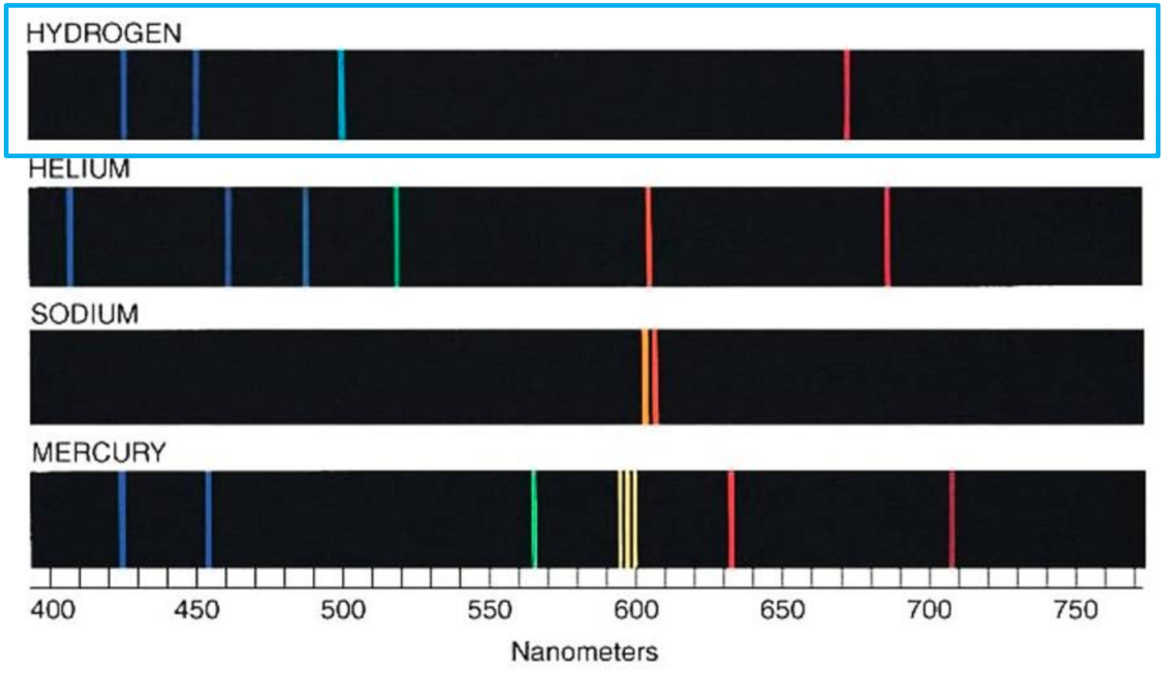
- **Nucleus charge**: $+Ze$ (where e = $1.60217662 \times 10^{-19}$ C is the elementary charge)

- **Ions**: charged atoms that have gained or lost an electron

Atomic Spectra

Each atom emits a unique light spectrum (fingerprint of an atom)

Observed emission spectra of atoms:



Researchers derived mathematical expressions to reproduce these observations

Atomic Spectra

Johannes Balmer determined the following expression for wavelengths of lines in the visible (400-800 nm) spectrum for hydrogen:

$$\boxed{\lambda = (364.6\ \mathrm{nm})\left(\frac{m^2}{m^2-4}\right)}$$

where m = 3, 4, 5,….

The **Rydberg-Ritz formula (**a more general expression for multiple elements across a wider wavelength range):

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)}$$

Conditions:
n_1 and n_2 are integers
$n_1 > n_2$
R: Rydberg constant (different for each element)

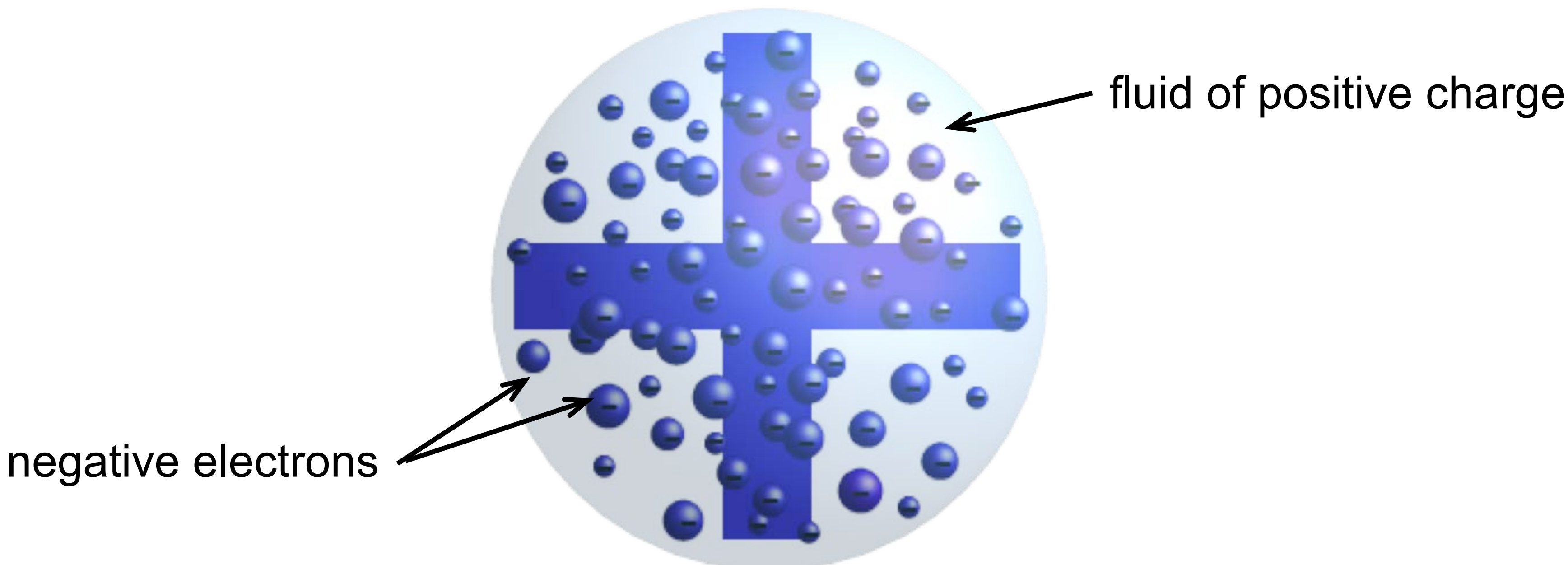
R_H = 1.097776 x 10^7 m^{-1} for hydrogen

The two are consistent for $R = R_H$, $n_2 = 2$, and $n_1 = m$

Models of the Atom:

Thompson's Plum Pudding Model

Electrons embedded in a fluid that has a distributed positive charge:

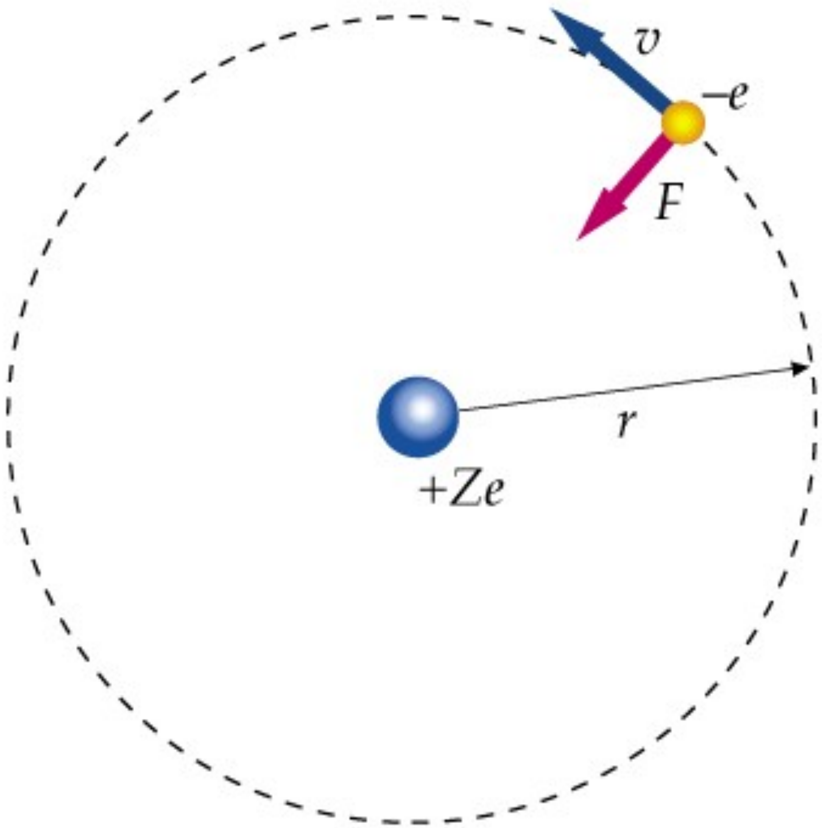


Disproved by his student (Rutherford) who showed that most of the mass of an atom is concentrated in the nucleus

The Bohr Model

Successfully predicted the observed spectra.

Structure similar to planets orbiting the sun: Electron moves in a circular/elliptical orbit around a positive nucleus:



ν: speed

$-e$: charge of electron

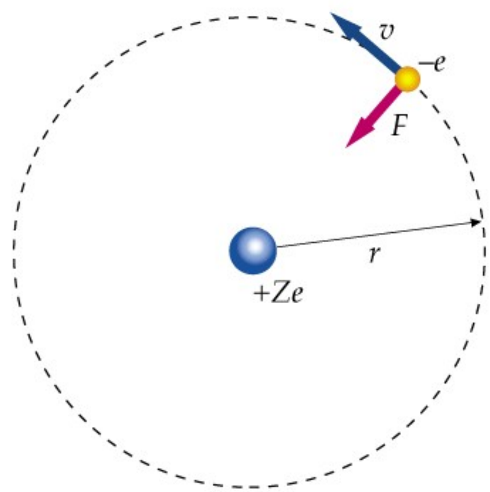
F: attractive electron force

r: radius

$+Ze$: charge of nucleus

The Bohr Model

Centripetal force acting on the orbiting electron is the electrostatic force of attraction (**Coulomb force**):



Attractive electrical force:

$$\boxed{F = \frac{k|q_1 q_2|}{r^2}}$$

where k = 8.99 x 10^9 N m^2 C^{-2} (Coulomb's constant)

$$\boxed{F = \frac{kZe^2}{r^2} = \frac{mv^2}{r}}$$

Keeps the electron in its orbit

Energy for a Circular Orbit

Potential energy (U) of electron of charge $-e$ at distance r from nucleus of positive charge Ze:

$$\boxed{U = \frac{kq_1q_2}{r} = \frac{k(Ze)(-e)}{r} = -\frac{kZe^2}{r}}$$

Kinetic energy (K) as function of r using Newton's 2nd Law ($F_{\text{net}} = ma$).
Set Coulomb attractive force equal to mass multiplied by centripetal acceleration:

$$\boxed{\frac{kZe^2}{r^2} = m\frac{v^2}{r}}$$

Multiply both sides by $r/2$:

$$\boxed{K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{kZe^2}{r}}$$

U and K vary inversely with r, and U is twice that of K:

UNIVERSITY OF LEICESTER

$$\boxed{U = -2K}$$

This is a general result for orbiting particles with $F \propto 1/r^2$

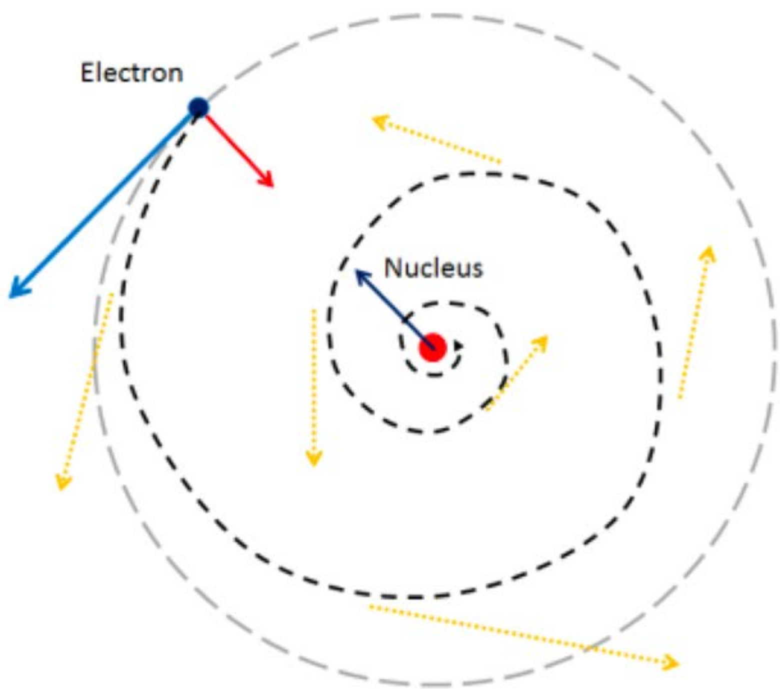
Therefore, total energy in a circular orbit is:

$$\boxed{E = K + U = -\frac{1}{2}\frac{kZe^2}{r}}$$

Why The Bohr Model Fails

Mechanically stable: Coulomb attractive force provides centripetal force to keep electron in orbit.

Electronically unstable: Electron must accelerate (change direction and velocity) when moving in a circle and so **emits electromagnetic radiation** of frequency equal to its motion, **loses energy**, and rapidly **spirals** into the nucleus.



[Source:
M. Strassler, 2013]

So, the atom could not exist using this model.

Bohr's Postulates

So Bohr made three assumptions (postulates) of his model:

1st Postulate. Electron in hydrogen atom can only move in certain non-radiating, circular orbits called **stationary states**

2nd Postulate. An electron can make a transition from a stationary state of higher energy, E_2, to a state of lower energy, E_1, leading to emission of a photon of frequency f.

$$\boxed{f = \frac{E_i - E_f}{h} = \frac{\Delta E}{h}}$$

where h = 6.626 x 10^{-34} m^2 kg s^{-1} (Planck's constant; also in units of J s)

We can use the total energy expression from earlier to obtain an expression for f in terms of r_f and r_i:

$$\boxed{f = \frac{E_i - E_f}{h} = \frac{1}{2}\frac{kZe^2}{h}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)}$$

Bohr's Postulates (contd)

3rd Postulate. Only those orbits are allowed for which angular momentum of an electron is an integer multiple of $h/2\pi$ (i.e. discreet values or $nh/2\pi$).

$$\boxed{\hbar = \frac{h}{2\pi} = 1.055\times10^{-34} Js = 6.582\times10^{-16}\ \text{eV s}}$$

where $\hbar$ is the reduced Planck's constant

The angular momentum of a circular orbit is mvr, so:

$$\boxed{mv_n r_n = n\hbar}$$

where n = 1, 2, 3,….
(the quantum number of the state)

Radii of Bohr Orbit

Derivation on board leading to:

$$\boxed{r_n = n^2 \frac{a_0}{Z}}$$

where r_n is the radius of orbit n

a_0 is the **first Bohr radius**
(orbital radius of the electron of a hydrogen atom that has $n = 1$)

$$\boxed{a_0 = \frac{\hbar^2}{mke^2} = \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.0529\ \mathrm{nm}}$$

Frequency of a Line

Obtain line frequency in terms of energy levels (or orbits) n

Derivation on board leading to:

$$\boxed{f = \frac{mk^2Z^2e^4}{4\pi\hbar^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

Use this and the relationship between frequency and wavelength ($f = c/\lambda$) and $Z = 1$ for hydrogen to obtain the Rydberg constant (R)

$$\boxed{R = \frac{mk^2e^4}{4\pi\hbar^3c} = \frac{me^4}{8\varepsilon_0^2h^3c} = 1.096776\times10^7\ \mathrm{m}^{-1}}$$

Value agrees with that obtained from spectroscopy

Atomic Energy Levels

Total mechanical energy of a hydrogen atom ($Z = 1$) is related to the radius of a circular orbit, so we can obtain an expression for the energy of orbit n for hydrogen

Derivation on board leading to:

$$\boxed{E_n = -\frac{1}{2}\frac{mk^2Z^2e^4}{n^2\hbar^2} = -Z^2\frac{E_0}{n^2}}$$

If $Z = 1$, E_n are the quantized allowed energies for hydrogen

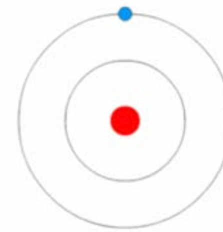
where: $n = 1, 2, 3, \ldots,$ and

$$\boxed{E_0 = \frac{mk^2e^4}{2\hbar^2} = \frac{1}{2}\frac{ke^2}{a_0} = 13.6\ \text{eV}}$$

Atomic Energy Levels (contd)

Transitions between allowed energies result in emission or absorption of a photon with frequency:

$$\boxed{f = \frac{E_i - E_f}{h}}$$



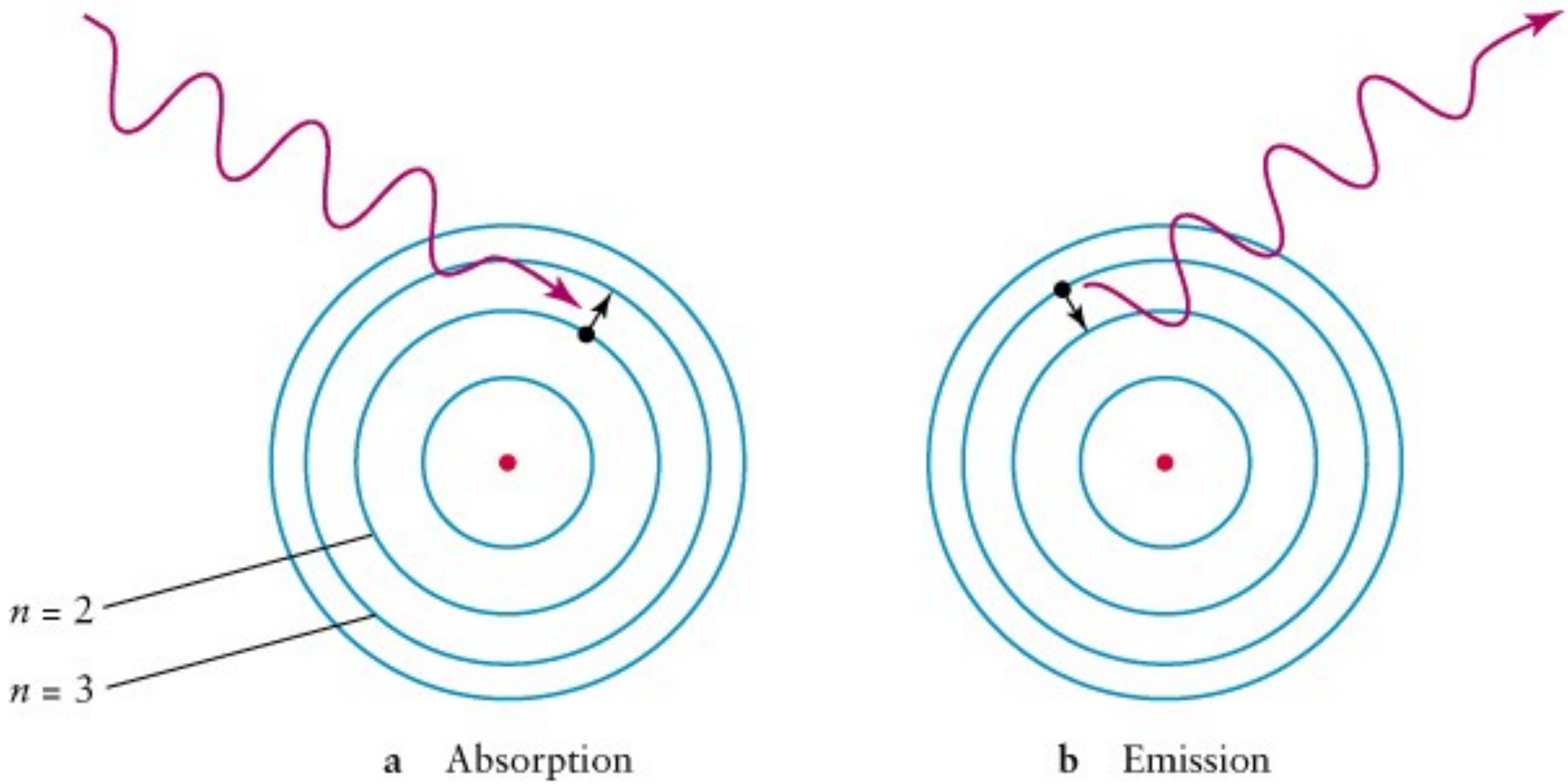
And of wavelength, λ, of:

$$\boxed{\lambda = \frac{c}{f} = \frac{hc}{E_i - E_f}}$$

where $hc = 1240 \text{ eV nm}$

f and λ of emitted and absorbed radiation are quantized (consistent with the observed line spectrum).

Quantized Energy Transitions

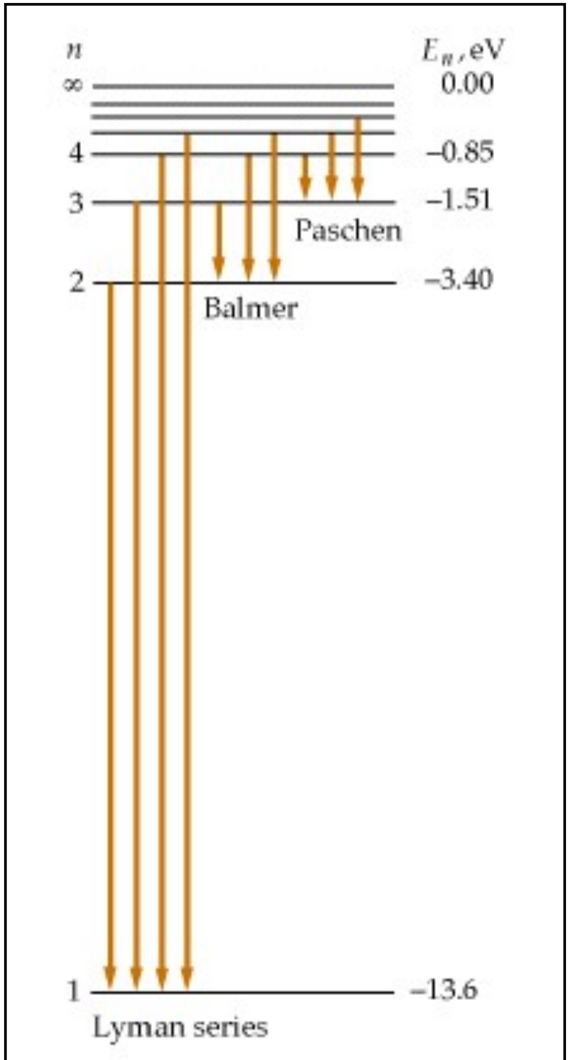


Electron transitions from lower to higher state (requires energy)

Electron transitions from higher to lower state (releases energy)

Energy Transitions of Hydrogen

Energy-level diagram



Arrows would be in opposite direction for absorption

$E_1 = -13.6\ \mathrm{eV}$
(ground state or lowest energy state of hydrogen)

As $n \to \infty$, $E_n \to 0$ (as $E_n \propto 1/n^2$)

Negative to indicate that energy is needed for electron to transition from lower to higher orbit

Energy-level diagram shows transitions from higher to lower states (emission lines):

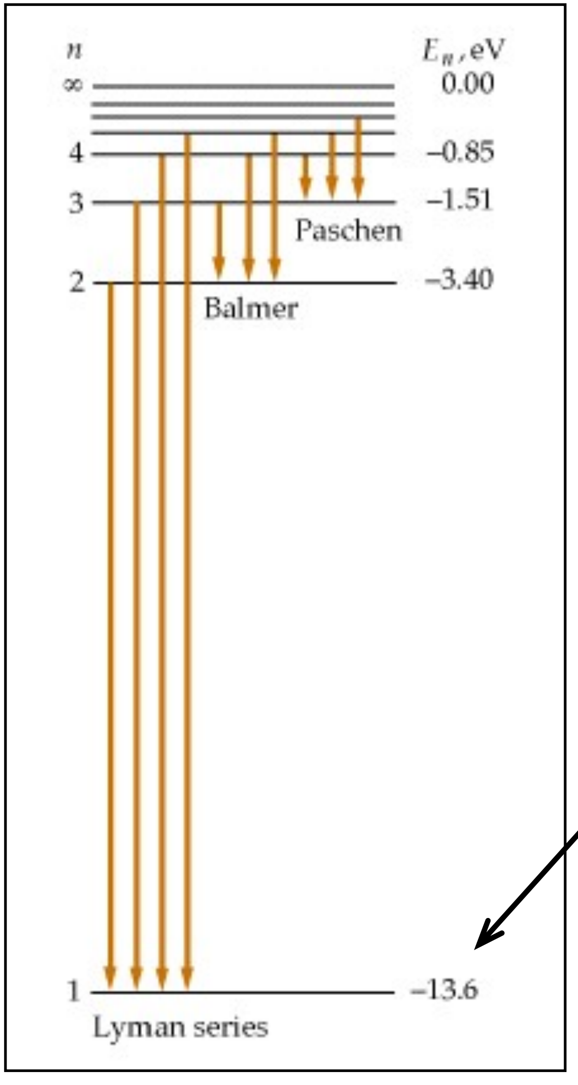
Balmer lines: $n_f = 2;\ n_i = 3, 4, 5, \ldots$

Paschen lines: $n_f = 3;\ n_i = 4, 5, 6, \ldots$

Lyman lines: $n_f = 1;\ n_i = 2, 3, 4, \ldots$

Electron Removal

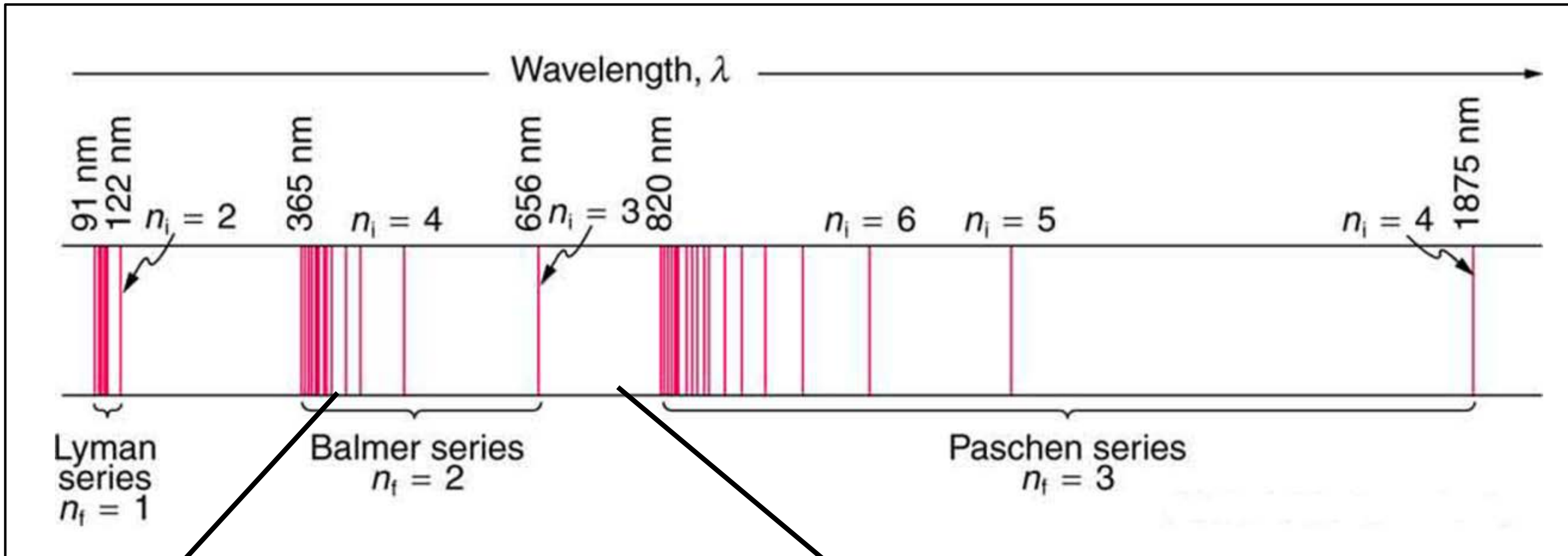
Energy-level diagram



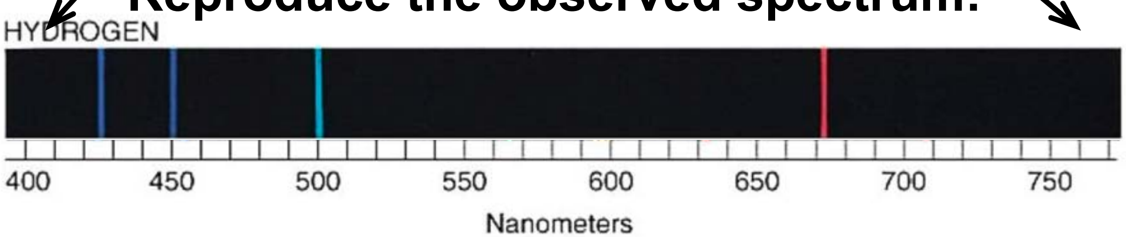
- **Ionization**: removal of one electron from an atom
- **Ionization energy** (or binding energy): minimum energy required to remove an electron

Energy Transitions

Able to reproduce the observed hydrogen spectrum with Bohr's model:



Reproduce the observed spectrum:



Issues with the Bohr Model

No justification for the assumption of stationary states

No justification for the assumption of quantization of angular momentum

Model not appropriate for atoms with more electrons and protons

The quantum-mechanical model resolves these issues